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An analysis of the introduction and book I of the text "High school geometry" by A.H. McDougall and R.S. Sheppard. 1930.

UNIVERSITY OF ALBERTA

AN ANALYSIS OF THE INTRODUCTION AND BOOK I OF THE TEXT " HIGH SCHOOL GEOMETRY " BY A.H. McDOUGALL AND R.S. SHEPPARD.

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BY

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CHAPTER I

INTRODUCTION

PURPOSE OF THE ANALYSIS

The text "High School Geometry" was introduced into the schools in Alberta in the fall of 1928. It was used in Geometry I for the first time that year and in Geometry II for the first time the following fall. Accompanying this change of texts in the schools, there has been a growing interest and change in the thought of teachers connected with the pedagogy of Geometry.

Briefly, current scientific and pedagogical thought contends that a course in Theoretical Geometry should be introduced by a course in Practical Geometry, in which the pupil forms his first geometrical concepts by the drawing of various figures accurately and by the making of certain measurements. According to the advocates of this theory such a course forms the stepping stone by which the pupil approachs the study of Theoretical Geometry.

This thesis is an attempt to analyse the text "High School Geometry", that is, the Introduction and Book I of the text, in order to present to the readers the nature of the development of the elementary concepts and the amount of emphasis given to the constitutent parts.

The study is of a comparative nature, as two books, Euclid's Elements[1] and Baker's Geometry[2] have been kept in

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1. Todhunter I. and Loney S.L., "The Elements of Euclid" Royal Holloway College, Egham, Surrey, England. 1899.

2. Baker, Alfred., "Geometry for Schools", University of Toronto, 1904.

mind. Baker's text was that which was used in the schools before the change to the present text. The comparison has been made with the Introduction and with the Propositions of Book I, but not with the exercises.

CHAPTER II

THE INTRODUCTION OF THE TEXT

Preliminary Definitions and Explanations.

The Introduction of the Text is in keeping with the general consensus of opinion that formal Geometry should be preceded by a practical course in inductive or experimental Geometry. The pupil receives his first ideas about the subject by making drawings of various types of figures. In fact the subject matter is presented in terms of figures. Table I gives a list of the definitions, showing those accompanied by figures as compared with the total number of definitions.

TABLE I

LIST OF THE DEFINITIONS SHOWING THOSE ACCOMPANIED BY ILLUSTRATIONS OR FIGURES

Definition of	Number of Illustrations
1. Point	2
2. Lines	3
3. Surface	0
4. Solid	0
5. Angle	1
6. Measurement of angles	1
7. Sign of an angle	3
8. Protractor	1
9. Adjacent angles	1
10. Perpendicular	1
11. Straight angle	1
12. Acute angle	1
13. Obtuse angle	1
14. Reflex angle	1
15. Complementary and Supplementary angles	0
16. Vert. opposite angles	1
17. Bisector of an angle	1
18. Parallel lines	2
19. Circle and parts of	1
20. Triangle, construction	1
21. Triangles, kinds of	6
22. Quadrilaterals, kinds of	9
Total	38

Hence it is seen that every concept that is presented to the pupil is accompanied by an illustration wherever it is possible. In each case references are made to the accompanying illustration so that the pupil is able to visualize the figure under discussion.

Symbols and Abbreviations

On page v of the text a complete list of the various symbols and abbreviations that are used throughout the text is given for constant and ready reference.

Instruments

The first section of the Introduction gives a list of the instruments needed. These are a hard sharp pencil, a ruler graduated in inches and fractions of an inch, and also in centimetres and millimetres, a pair of compasses, a protractor a set square, and a supply of tracing paper. The pupils are told in the preface of the text that "all drawings should be drawn neatly and accurately with geometrical instruments" and in the Introduction that "an accuracy within one or two per cent can be obtained with ordinary instruments".

Points, Lines and Surfaces.

The first concept of the straight line that the pupil receives is developed by the folding of a sheet of paper. The fold is said to be a straight line.

The first concept of the point is obtained by noting when two straight lines are drawn they intersect at a point. A point is defined as that " which has position but not magnitude". It has"neither length nor breadth nor thickness."

The authors do not mention that the "ends of lines are points" nor do they mention that "the term point is also applied to the division, without length, which separates one portion of a line from another portion of the same line." Todhunter's Euclid makes mention of the first concept and Baker's text makes mention of both concepts.

A line is defined as " that which has length, but not breadth nor thickness". The line which we draw "only roughly represents the idea". Straight, curved and broken lines are distinguished. A straight line is defined as that which "has the shortest distance between two points". "Direction" and "distance" are not explained. A figure of a straight line, of a curved line, and of a broken line aids in the distinction.

A surface is defined as " that which has length and breadth but no thickness". A sheet of tissue paper roughly represents a surface, or rather two surfaces separated by the substance of the paper. "The boundary between two parts of space is a surface." "Surfaces may be either plane or curved."

A plane surface is defined as "that in which any two points being taken, the straight line joining them lies wholly in that surface." The force of the word "any" is pointed out to the pupil.

A solid is defined as "that which has length, breadth and thickness".

A figure is defined as "any combination of points, lines, surfaces and solids".

Geometry is defined as " the science which investigates the properties of figures and the relations of figures to one another". Plane Geometry and Solid Geometry are distinguished "In plane Geometry the figure, or figures, considered in each proposition are confined to one plane, while the Solid Geometry treats of figures the parts of which are not all in the same plane." " Plane Geometry is also called the Geometry of Two Dimensions (length and breadth) and Solid Geometry is called the Geometry of Three Dimensions (length, breadth and thickness).

The pupil is shown how to measure the distance between two points, by means of compasses and a ruler, that is, the measurement of a straight line, although no mention is made of the fact that the two points are at the ends of a straight line. The pupil is also shown how to cut from the greater of two straight lines a part equal to the less, by means of compasses. These last two problems are given in Todhunter's Euclid I, 2 and I, 3.

The exercises, which follow, give the pupil practice in drawing straight lines, in bisecting them, in drawing lines from a point to a line, and in finding ppints in a line which are a certain distance from a given point. The pupil is also led to discover that all points on the circumference of the circle are equidistant from the centre. He is shown that a triangle may be drawn by using the ends of the base as centres, and by means of arcs with radii of given length from these centres, locating the vertex and completing the triangle. These last four exercises are introductory, as no reference

has been made up to this part of the text to either circles, or triangles. At the end of this chapter a summary of the exercises is given.

Angles.

An angle is defined as follows : "Let a straight line AB rotate about A, in the plane of the paper, from a position AB to another position AC. The amount of turning which the line has done in rotating about A from the position AB to the position AC is called an angle." This definition is accompanied by a figure.

The vertex A, and the arms AB and AC of the angle are mentioned. The pupil is also told that in naming an angle that "the letter at the vertex must be the middle one in reading the angle". Sometimes the single letter at the vertex is used to denote the angle when there can be no doubt as to which angle is meant.

The text also points out that by making a tracing of an angle and placing this on another angle, vertex to vertex, and the arm of one coinciding with an arm of the other, that a comparison of the size of the two angles may be obtained. This concept paves the way for the idea of superposition which the pupil encounters in Book I.

By rotating one arm of an angle in either direction the pupil obtains the idea that there is a positive and a negative direction, and hence a positive and negative sign; and also obtains the idea that " the magnitude of an angle depends altogether on the amount of rotation, and is quite independent of the length of its arms".

The pupil is shown how to use the protractor to measure angles in degrees, and how to make angles containing a given number of degrees. Ten exercises follow in the measurement and the making of angles. One of these leads the pupil to discover experimentally that the angle sum of a triangle is 180°.

Further definitions and explanations about kinds of angles follow, each accompanied by a figure. "Adjacent angles" are defined as angles which "have the same vertex and a common arm, the remaining arms being on opposite sides of the common arm". Right angle and perpendicular are defined as: "when one straight line stands on another so as to make the adjacent angles equal to one another, each of the angles is called a right angle and each line is said to be perpendicular to the other." A table of measurement of angles is given mentioning right angle, degrees, minutes and seconds.

The other definitions given are :

"Straight angle": "If one arm OB of an angle turns until it makes a straight line with the other OA, the angle so formed is called a straight angle."

"Acute angle": "An angle which is less than one right angle is said to be acute."

"Obtuse angle": "An angle which is greater than one right angle, but less than two right angles is said to be obtuse."

"Reflex angle": "An angle which is greater than two right angles but less than four right angles is said to be reflex."

A series of five exercises on the drawing of diagrams to scale follows this section. These exercises necessitate the accurate drawing of lines and angles.

Supplementary angles are defined as two angles whose sum is two right angles. Each angle is said to be the supplement of the other. Complementary angles are defined as two angles whose sum is one right angle. Each angle is said to be the complement of the other. A series of nine exercises on supplementary and complementary angles follows this section.

Certain simple and obvious theorems on angles enter naturally at this part of the Introduction. These are the first theorems that the pupil encounters in the text. Baker, in his section on angles, points out that the theorems are true but does not attempt any proofs of a rigorous nature. This text in contrast offers proofs but these proofs are not as rigorous in style as those offered by Todhunter's Euclid. The following is a list of the theorems offered by this text, with references to Euclid. The same theorems are found on page 15, 16 of Baker :

a. " The angles which one straight line makes with another on the same side of that other are together equal to two right angles." (Euclid I, 13)

b. "If two adjacent angles are together equal to two right angles, the exterior arms of these angles are in the same straight line." (Euclid I, 14)

c. "If any number of straight lines meet at a point, the sum

of the successive angles is equal to four right angles." (Euclid I, 15, Cor. 2)

d. " Each of the angles formed by two intersecting straight lines is equal to the vertically opposite angle." (Euclid I,15)

At the end of this section the pupil is given the concept of a bisector by means of a hypothetical construction.

Nine fairly simple exercises follow, based on the theorems. In the last exercise, the pupil is led to see some of the properties of parallelism, although no mention is made of the fact that the pupil is dealing with two parallel lines, cut by a transversal.

Parallel Lines.

Parallel straight lines are said to be "those which have the same direction throughout their entire length", or "two straight lines in the same plane which do not meet when produced for any finite distance in either direction."

Euclid gives us an axiom of parallelism in axiom 12, but no mention is made of such an axiom here. The treatment of parallelism is similar to that of Baker's. Deviation from a common line is explained, showing that parallel lines deviate by the same amount from any line that intersects them. The following tests of parallelism are give with explanations:

a. "If a transversal cut two parallel lines the exterior angle equals the interior and opposite angle on the same side of the transversal."

b. "If a transversal cut two two other straight lines and makes the exterior angle equal to the interior and opposite

angle on the same side of the transversal the two straight lines are parallel."

An exercise of two problems follows this discussion developing chiefly the concept of equal angles made by the transversal cutting parallel lines.

The Circle.

A short discussion is given here on the circle, explaining the names of the various parts. A fuller explanation of the concept of the circle is given in the section on loci in the the introduction to Book III.

A circle is said to be "a figure bounded by a curved line, called the circumference, and is such that all straight lines drawn from a certain point within the figure called the centre, to the circumference are equal to one another." These lines are called radii. " A straight line joining two points on the circumference is called a chord." A chord through the centre is called a diameter. A part of the circumference is called an arc. The concept of a part of the circumference, or an arc subtending an angle, and that of a semi circle is explained. A figure is given.

This section on the circle is very similar to the section in Todhunter's Euclid on the circle, except that Euclid mentions the concept of "segment", which is omitted in this text. Baker's discussion in his introduction is very similar to this discussion also, except that Baker mentions "segment", "secant", and "sector" which are omitted here.

No exercises are given on the circle in the Introduction.

However use is made in Book I of the various concepts whish are developed here.

Rectilineal Figures.

A rectilineal figure is defined as one "formed by straight lines". A triangle is defined as "a figure formed by three straight lines which intersect one another." "The three points of intersection are called the vertices of the triangle," and the "lines between the vertices are called the sides of the triangle".

The pupil is shown how to construct a triangle with sides of given length. Baker gives this concept as his first Proposition in Book I. Euclid gives this as a problem in I,22. The method of construction is similar in each case to that given by this text.

A series of twenty exercises on the triangle, and its construction, as well as the concepts developed earlier in the Introduction, follows this section. Some of the exercises anticipate concepts which are developed later. For example, in some of the exercises the pupil discovers for himself that the angle sum of a triangle is 180°.

Triangles are classified according to the number of sides and according to angles as follows :

a. "Equilateral" when all the sides are equal.

b. "Isosceles" when two of the sides are equal.

c. "Scalene" when all three sides are unequal.

d. "Right-angled" when one of the angles is a right angle.

e. "Obtuse-angled" when one of the angles is obtuse.

f. "Acute-angled" when all three of the angles are acute.

In a right-angled triangle the side opposite the right angle is called the hypotenuse.

An exercise of five problems follows this section developing the ideas given about the kinds of triangles.

The theorem that "the sum of the angles of a triangle is two right angles" is carefully proven here, and the accompanying theorem that "the exterior angle of a triangle is equal to the sum of the two interior and opposite angles, and therefore is greater than either of them" is shown to be true. Baker gives these two theorems in his introduction, but does not prove them in as Euclidean a manner as this text does. Baker also stresses that "any two angles of a triangle are together less than two right angles", which is mentioned by this text. Euclid proves the first two theorems in I, 32, and the third in I, 17. The proofs by Euclid of the first two theorems are practically the same as the proofs given by this text.

A series of seven exercises follows, which require the drawing of diagrams to scale. The theorems given in the preceding section are used in these exercises.

Quadrilaterals.

Quadrilateral, diagonal, rhombus, square, parallelogram, rectangle, and trapezium are defined here. The definitions are the same as those given by both Baker and Euclid in their introductions. Polygon, equilateral polygon, equiangular polygon, regular polygon, pentagon, hexagon and octagon are defined. The pupil is also shown how to construct a quadrilateral

given three sides and the two included angles. In the series of fifteen and two problems which follow these sections, the pupil is required to construct various kinds of rectilineal figures.

Geometrical Reasoning.

The section on Geometrical Reasoning is a little fuller in this text, and is organized better than the similar section in Todhunter's Euclid, and in Baker's text.

The section points out the distinction between the Practical and Theoretical methods of investigating the properties and relations of figures. It also shows that the Theoretical method has certain advantages over the Practical method, as measurements are never exact and in many cases cannot be made directly.

The Theoretical method starts with certain simple statements called axioms, the truth of which is either self evident or assumed, and the consequent statements follow with absolute certainity, if there is no flaw in the reasoning.

The axiom of superposition, by which figures are proved to be congruent, is stated here as : "A figure may be, actually or mentally, transferred from one position to another without change of form or size."

Propositions.

"In general, a proposition is that which is stated or affirmed for discussion". Propositions are of two kinds: theorems, i.e. statements of some geometrical truth; and

problems, i.e. statements of some required geometrical construction. The text makes clear just what the General Enunciation, Particular Enunciation, Construction, Proof, Hypothesis, and Conclusion of a Proposition are.

Axioms.

The following axioms are listed in addition to the one mentioned above. The one above is not numbered as one of this list. Each of the following is illustrated by simple examples.

1. Things that are equal to the same thing are equal to each other.
2. If equals be added to equals, the sums are equal.
3. If equals be taken from equals, the remainders are equal.
4. If equals be added to unequals, the sums are unequal, the greater sum being obtained from the greater unequal.
5. If equals be taken from unequals, the remainders are unequals, the greater remainder being obtained from the greater unequal.
6. Doubles of the same thing, or of equal things, are equal to each other.
7. Halves of the same thing, or of equal things, are equal to each other.
8. The whole is greater than its part, and equal to the sum of the parts.
9. Magnitudes that coincide with each other, are equal to each other.

The axioms listed above are almost the same as those

given by Baker and Euclid in their texts. This text has added the last phrase in axioms 4, 5, and 8. Baker and Euclid also give the following axioms:

1. Two straight lines cannot enclose a space.
2. All right angles are equal to each other.

Baker does not list the second of these, but he makes mention of it in connection with his list. This text does not mention that there is an axiom of parallelism here, although mention is made of the tests in the section on parallelism. Euclid in his text gives his famous axiom 12, and Baker gives a modification of it as : " If a straight line fall on two parallel lines, it makes the exterior angle equal to the interior and opposite angle." This is one of the tests given by this text.

Postulates.

In the Introductions of the texts in Geometry there is usually included a list of three postulates. The text gives these in Book I, following Proposition IV, before the problems on constructions. At that point in the text the pupil is told that "in Theoretical Geometry the use of instruments in making constructions is generally restricted to an ungraduated straight edge and a pair of compasses. With these instruments we can :

1. Draw a straight line from one point to another.
2. Produce a straight line.
3. Describe a circle with any point as its centre and radius equal to any given straight line.

4. Cut off from one straight line a part equal to another straight line."

The first three postulates are given by both Baker and Euclid. The fourth postulate is not given by Euclid, and is only mentioned as possible by Baker.

Exercises.

In the Introduction there are 92 exercises. Table II and Table III show the drill given by these and the distribution of the exercises. In Table II these are listed according to the concept drilled in each. In classifying these it was found in some cases that a greater part of the drill was given to one concept and that other concepts entered to supplement this drill. In such cases the exercises were listed as drill of that important concept. In this way a check was obtained, as the total number of units of drill equalled the total number of exercises.

TABLE II

THE DISTRIBUTION OF THE EXERCISES IN THE INTRODUCTION

Concept Drilled	Number of Exercises
1. Points, lines	5
2. Angles	33
3. Parallel Lines	3
4. Circle	1
5. Triangles	37
6. Quadrilaterals	13

TABLE III.

NUMBER OF UNITS OF DRILL ON EACH CONCEPT IN THE INTRODUCTION

Concept Drilled in Introduction	Place relative to Section		
	Number Prior	Immediately Following	Later in Introd.
1. Points, lines	0	5	0
2. Angles, measurement of	0	10	0
Angles, scale diagrams	0	4	0
Angles, kinds of	0	9	0
Angles, theorems on	0	8	2
Angles, total	0	31	2
3. Parallel Lines	1	2	0
4. Circle	1	0	0
5. Triangles, construction	3	19	4
Triangle, scale diagram	4	7	0
Triangles, total	7	26	4
6. Quadrilaterals, construction of	0	13	0
Total Number	9	77	6

Table II and III show clearly the distribution of the exercises. Apparently the most important concepts drilled are those of angles and triangles. Quadrilaterals rank third, but it must be remembered that this topic includes a study of many kinds of figures. Points and angles, although receiving apparently little drill, are used in the drill on angles and triangles of which they form a natural part.

Most of the drill appears immediately following the

section which develops the concept. Peculiarly there are more exercises appearing before the sections developing the concepts in a formal manner than there are exercises following these sections. This is in keeping with the experimental nature of the Introduction, as the pupil is led to discover these truths for himself before they are presented to him in a formal manner.

It might be noted in passing that the exercises on the triangle are quite varied, embracing in many cases other concepts such as angles. In the exercises on triangles, the work in Book I on the congruence of triangles is anticipated in part as the pupil deals with some of the cases experimentally. For example in exercise 15 on page 23 the pupil deals with the case proven in I, 1 of the text; in exercise 16 on the same page he deals with I,3, and in exercise 17 with I, 4. These are the three cases of the congruence of triangles which are given the most drill as will be shown in chapter IV of this thesis.

Conclusions.

1. The Introduction of the text is nearly the same as the Introduction of Baker's text. They differ in part, naturally, but the concepts presented are much the same. Mention will be made of this point again in Chapter V where a fuller summary comparison is made of the texts.

2. The introduction of this text is much fuller than the Introduction to Todhunter's Euclid, so that the

comparison should include part of Euclid's Book I. On the whole the comparison shows that Euclid has been followed in material if not in arrangement. The chief difference between the two lies probably in the aim at the back of each Introduction. It is generally agreed that Euclid wrote his text for adult's, that is, people of maturity, whereas this text was written for pupils of school age.

3. This text was written so that an experimental presentation of the Introduction could be made. This is shown particularly in the types of exercises. For example, the pupil is led to see experimentally that the cases of the congruence of triangles are apparently true; and that the angle sum of a triangle is apparently 180°, before he comes to the proof of these. Other examples of this are mentioned in this chapter.

4. Table II and Table III show the distribution of the drill given by the exercises. The most important concept, if the number of exercises is a reflection of the importance, is that of triangles, with that of angles coming next.

CHAPTER III

BOOK I TRIANGLES

Introduction

Book I deals with the Congruence of Triangles, Constructions and Inequalities in Triangles. The distribution of the Propositions is shown by Table IV.

TABLE IV

DISTRIBUTION OF THE PROPOSITIONS OF BOOK I ACCORDING TO THE CONCEPT PRESENTED.

Concept Presented	Propositions
A. Theorems	
1. Congruence of triangles	1,3,4,13
2. The isosceles triangle	2, 12.
3. Inequalities in one triangle	10, 11
4. Inequalities in two triangles	14, 15
B. Problems	
1. Bisectors	5, 7,
2. Perpendiculars	6, 8.
3. Construction of an angle	9.

Table IV shows an arbitrary topical grouping of the Propositions. Each of these will be dealt with briefly and separately in this chapter. In some cases there are changes of order from that given by Euclid and by Baker in their texts. To make clear the relative positions of the Propositions in each of these texts, corresponding references are made to Euclid and to Baker, in stating each Proposition of this text. Also similarities and differences are pointed out in the discussion of the Propositions.

In this Book there are 177 exercises. These appear in sets after thirteen of the fifteen Propositions and in the miscellaneous exercises at the end of the Book. These exercises are discussed in the chapter.

Propositions.

The classification of the Propositions given in Table IV is not according to the sequence of the Book, as will be noted by the numbering of the Propositions. To maintain this sequence, the Propositions are discussed in the order in which they are presented in the text. This discussion consists of the General Enunciation, a reference to Euclid and to Baker, and a note on the nature of the proof.

Proposition I Theorem. : " If two triangles have two sides and the contained angle of one respectively equal to two sides and the contained angle of the other, the triangles are congruent." (Euclid I, 4 ; Baker I,6)

The method of proof is that of superposition, although no mention is made of the axiom of superposition.

Proposition 2 Theorem. : " The angles at the base of an isosceles triangle are equal to each other."
(Euclid I, 5 ; Baker I,2)

The method of proof in this text is the drawing of the bisector of the vertical angle, and the proving of the congruence of the resulting triangles by Proposition I, and hence the equality of the angles at the base. Baker, in his text reverses the triangle and by the method of superposition proves that the angles at the base are equal to each other.

Baker includes a corollary proving that if the angles are equal, and that if the equal sides be produced, the angles on the other side of the base will also be equal to each other. Baker lists a second corollary which points out that the angles of an equilateral triangle are equal to each other, Euclid gives the first mentioned theorem and Baker's first corollary as his Proposition and lists Baker's second corollary as his first corollary. Euclid makes use of his theorem I, 4 in proving this Proposition.

Proposition 3 Theorem : " If two triangles have the three sides of one respectively equal to the three sides of the other, the two triangles are congruent."(Euclid I,8; Baker I,4)

The proof given here is the same as the proof given by Baker, that is, applying the base of one triangle to the base of the other and allowing it to take the position on the other side of the base, joining the vertices, and proving the resulting triangles congruent. Euclid uses the method of superposition.

Proposition 4 Theorem : " If two triangles have two angles and a side of one respectively equal to two angles and the corresponding side of the other, the triangles are congruent." (Euclid I, 26 Case I ; Baker I, 7)

The proof of this Proposition is based on the axiom of superposition in both this text and in Baker's text. Euclid does not use this axiom.

Proposition 5 Problem : " Bisect a given angle."
(Euclid I,9 ; Baker I,9)

The method of bisecting the angle and the proof are the same in all three texts.

Postulates.

At this point in the text the Postulates are introduced, as mentioned in Chapter II. A discussion of these is given there. It should be noted in passing that this text does not attach the name of Postulates to these.

Proposition 6 Problem : " Draw a perpendicular to a given straight line from a given point in the line."
(Euclid I, 11 ; Baker I, 11)

The method of drawing the perpendicular and the proof are the same in all three texts.

Corollary

Corollary is defined here as " Sometimes when a proposition has been proved, the truth of another proposition follows as an immediate consequence of the former; such a proposition is called a corollary."

Right Bisector.

Right Bisector is defined here as " a straight line which bisects a line of given length at right angles."

Proposition 7 Problem : "Bisect a given straight line."
(Euclid I, 10 ; Baker I, 10)

The method of drawing the bisector and the proof are the same in this text as that used by Euclid. Baker uses a more complicated form of construction and proof.

Corollary to Proposition 7 : "The bisector is a right bisector." This is not pointed out by Euclid or by Baker in connection with this Proposition.

Median.

Median is defined here as "The straight line drawn from a vertex of a triangle to the middle point of the opposite side".

<u>Proposition 8 Problem</u> : " Draw a perpendicular to a given straight line from a given point without the line."
(Euclid I, 12 ; Baker I, 12)

The method of construction and proof used in this text is a little different from that given by Euclid, in that Euclid bisects the base of the triangle which is drawn, while this text bisects the vertical angle of the same triangle. Baker uses a more complicated form of construction and proof similar to one he used for Proposition 7 of his text.

<u>Proposition 9 Problem</u> : "Construct an angle equal to a given angle."   (Euclid I, 23 ; Baker I, 5)

The method of construction and proof is the same in this text as that given by both Euclid and Baker.

<u>Proposition 10 Theorem</u> : "If one side of a triangle be greater than another side, the angle opposite the greater side is greater than the angle opposite the less side."
(Euclid I, 18 ; Baker I, 13)

The method of proof used by Euclid and by Baker for this Proposition is the same as that used here.

<u>Proposition 11 Theorem</u> : (Converse of I, 10): " If one angle of a triangle be greater than another angle of the same triangle, the side opposite the greater angle is greater than the side opposite the less." (Euclid I, 19 ; Baker I, 14)

The method of proof used here is the same as that used by Euclid and is called "the indirect method of demonstration". Baker uses a direct method, which is somewhat similar to the one he used in his Proposition 13.

Converse Propositions.

This text explains the indirect method of demonstration carefully to the pupil and points out that such a method is commonly used for proving converse propositions. Converse Propositions are defined here as : " When two propositions are such that the hypothesis of each is the conclusion of the other, they are said to be converse propositions."

Proposition 12 Theorem (Converse of I,12) : "If two angles of a triangle are equal to each other, the sides opposite these equal angles are equal to each other."
(Euclid I, 6 ; Baker I, 3)

This text, like Euclid, uses the indirect method of demonstration. Baker uses the method of reversing the triangle and superposing it.

Proposition 13 Theorem (The ambiguous case of the congruence of triangles) : " If two triangles have two sides of one respectively equal to two sides of the other and have the angles opposite one pair of equal sides equal to each other, the angles opposite the other pair of equal sides are either equal or supplementary." (Baker I, 8)

The method of proof given by Baker is the same as that used in this text. Euclid does not give this as a separate proposition in his Book I.

Corollary to Proposition 13 "If two right-angles triangles have the hypotenuse and a side one respectively equal to the hypotenuse and a side of the other, the triangles are congruent." (Baker I, 8 cor.)

The method of proving this corollary in this text is the same as the method used by Baker.

Sequence.

At this point in the text, a sequence is established between the cases of the congruence of triangles, by pointing out to the pupil that there are five general cases. These cases, with references to the Propositions in which they are proven, are:

1. Two sides and the contained angle. Triangles congruent. Proposition I, Book I.
2. Three sides. Triangles are congruent. Proposition 3.
3. Two angles and a side. Triangles congruent. Proposition 4.
4. Two sides and an angle opposite one of them. In this case the triangles are congruent if the angle is opposite the greater of the two sides, but if the angle is opposite the shorter of the two sides, they are not necessarily congruent. The first case is shown by Exercise 2 on page 62. The second case is proven by Proposition 13.
5. Three angles. The triangles are not necessarily congruent. The student is referred to exercise 6 on page 62.

Proposition 14 Theorem : " If two triangles have two sides on one respectively equal to two sides of the other, but the

contained angle of one greater than the contained angle of the other, the base of the triangle which has the greater angle is greater than the base of the other."
(Euclid I, 24 ; Baker I, 16)

The method of proof used by this text is similar to the method of proof used by both Euclid and Baker.

Proposition 15 Theorem (Converse of I, 14) : "If two triangles have two sides of one respectively equal to two sides of the other but the base of one greater than the base of the other, the triangle which has the greater base has the greater vertical angle." (Euclid I, 25 ; Baker I, 17)

The method of proof used here is the indirect method of demonstration or the method of "exhaustion", as the proposition is proven by showing that only the hypothesis and no other conclusion can be the right one. This is the method that is used by both Baker and Euclid in their texts.

Conclusion.

References are made throughout this chapter to Todhunter's Euclid and to Baker's text in the brief comparative study that is made of the Propositions. To draw conclusions from this study it will be convenient to note first the sequence of the Propositions in the three texts. Table V shows in a summary the Propositions in Euclid and in Baker corresponding to the Propositions of this text. In the table the comparison is made by giving the number of the Proposition in Book I in each case.

TABLE V

COMPARISON OF THE SEQUENCE OF CORRESPONDING PROPOSITIONS IN EUCLID'S, BAKER'S AND McDOUGALL AND SHEPPARD'S TEXTS. PROPOSITIONS OF BOOK I SHOWN BY NUMBER OF PROPOSITION.

McDougall and Sheppard	Euclid	Baker
Proposition 1	4	6
Proposition 2	5	2
Proposition 3	8	4
Proposition 4	26	7
Proposition 5	9	9
Proposition 6	11	11
Proposition 7	10	10
Proposition 8	12	12
Proposition 9	23	5
Proposition 10	18	13
Proposition 11	19	14
Proposition 12	6	3
Proposition 13	..	8
Proposition 14	24	16
Proposition 15	25	17

On the first glance at the Table much of Euclid's Book I dealing with triangles appears to have been omitted in this text, but it must be remembered that Propositions 1, 2, 3, 13, 14, 15, 16, 17, 22 were dealt with in a simple manner in the Introduction of this text, as they were in Baker's text. In comparing with Baker it must be remembered that Baker's Proposition I is dealt with in the Introduction to this text. The only omissions then are Euclid's Propositions 20, 21 and Baker's Proposition 15. These Propositions deal with relations of sides of a triangle. The theorems in Euclid are :

"Any two sides of a triangle are together greater than the third side." (Euclid I, 20)

"If from the ends of the side of a triangle there be drawn two straight lines to a point within the triangle, these shall be less than the other two sides of the triangle but shall contain a greater angle." (Euclid I, 21)

Baker's Proposition 15 is the same as Euclid's Proposition 21.

In conclusion, then, most of the material presented in both Euclid's text and in Baker's text is given in this text. The method of proof in each case is much the same as that used by Euclid, i.e. Todhunter's Euclid, and By Baker. The sequence in this text is different. A fuller summarizing comparison is made of these texts in Chapter V of this thesis.

CHAPTER IV

THE EXERCISES OF BOOK I

Introduction.

In Chapter III in the discussion of the Propositions of Book I, no mention was made of the exercises of Book I, although they are naturally connected with the Propositions. It will be the purpose of this chapter to present in various ways a classification of these exercises.

Distribution.

Table VI shows the distribution of the exercises as they appear in the Book.

TABLE VI

DISTRIBUTION OF THE EXERCISES OF BOOK I LISTED BY PROPOSITIONS

Proposition	Number of exercises following
1	10
2	7
3	3
4	7
5	4
6	8
7	14
8	4
9	11
10	0
11	13
12	10
13	6
14	0
15	8
Miscellaneous Exercise	70
Total	177

Table VI shows the distribution of the exercises to be fairly even in Book I. There are no exercises after Propositions 10 and 14. These Propositions are drilled in the exercises which follow Propositions 11 and 15 respectively, as will be noted by the presentation which follows.

Classification by Propositions.

In order to determine the amount of drill each Proposition received, each of the 177 exercises was worked, and notes were made as to the Propositions which were drilled in each exercise. In such a classification it was found that in some exercises more stress was placed on some Propositions than on others. It was also found that one of several Propositions could often be used to obtain the solution of the exercise. In order to make the classification valid and definite a criterion was established.

Briefly the criterion was as follows. If a Proposition appeared to be used more than others, or if it appeared to be the most important Proposition used in the exercise, it was listed as a major Proposition for that exercise. If two or more Propositions appeared to be used equally or to be equally important, these were listed as major Propositions. Other Propositions used as a means to an end in the exercise were listed as minor Propositions. Propositions, chiefly Problems, used in the construction of the figure were listed separately. These notes were made separately for each exercise and were summed up.

If two or more Propositions were equally applicable

to the exercises or to one exercise as alternative methods of proof, the Proposition was used which appeared to have the easier application, from a pupil's point of view.

The classification was summed up in the following figures, which show the amount of drill given each Proposition.

Figures Showing Distribution of Drill.

DISTRIBUTION OF THE DRILL BY EXERCISES IN BOOK I.

HISTOGRAMS SHOWING THE AMOUNT OF DRILL GIVEN EACH PROPOSITION AFTER EACH OF THE PROPOSITIONS OF BOOK I AND IN THE MISCELLANEOUS EXERCISE. HORIZONTAL AXIS SHOWS THE PROPOSITIONS.

Legend :

A unit of Major Drill

A unit of Minor Drill

Proposition used in construction.

FIGURE I. PROPOSITION I.

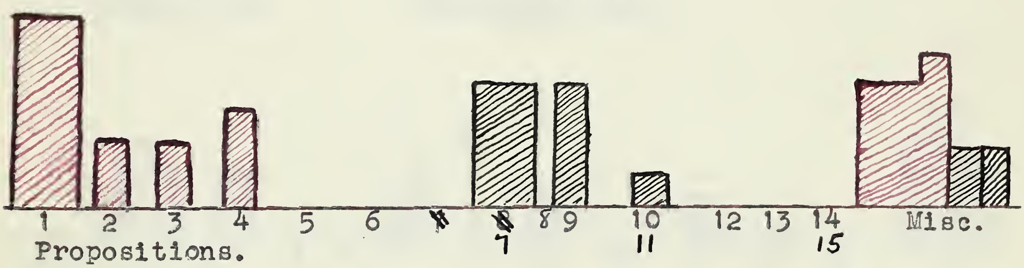


Figure I shows the drill given Proposition I. According to the notation used, there are ten units of major drill following Proposition 1, two after 2, 2 after 3, three after 4, and 13 in the miscellaneous exercises. There are eight units of minor drill after Proposition 7, four after 9, and one after 11. Proposition 1 is used in two exercises in the miscellaneous exercises for the construction.

FIGURE II. PROPOSITION II.

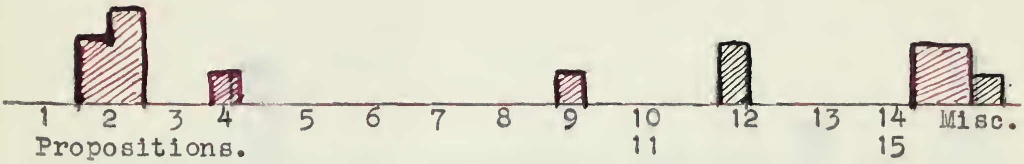


Figure II shows the drill given Proposition II. According to the notation used, there are five units of major drill after Proposition 2, one after 4, one after 9, and four in the miscellaneous exercise. There are two units of minor drill after Proposition 12 and one in the miscellaneous exercise. It will be noted that Propositions 10 and 11 are grouped and that Propositions 14 and 15 are grouped on the horizontal axis.

Figure III. Proposition III.

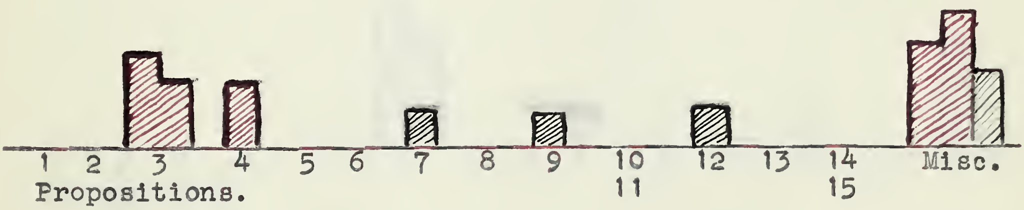


Figure III shows the distribution of the drill given Proposition III by the exercises of Book I. According to the notation used, there are five units of Major Drill after Proposition III, two after 4, and seven in the miscellaneous exercise. There is one unit of Minor Drill after Proposition 7, one after 9, and one after 12. The Proposition is used in two exercises in the miscellaneous exercise.

Similar explanations apply to the other figures.

Figure IV. PROPOSITION IV.

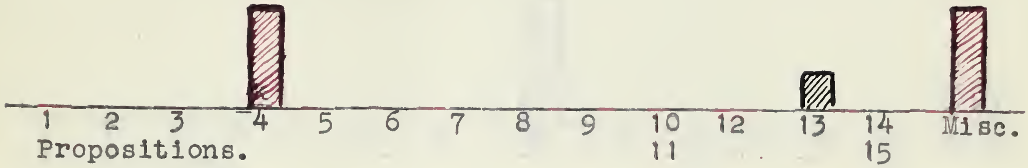


FIGURE V PROPOSITION V.

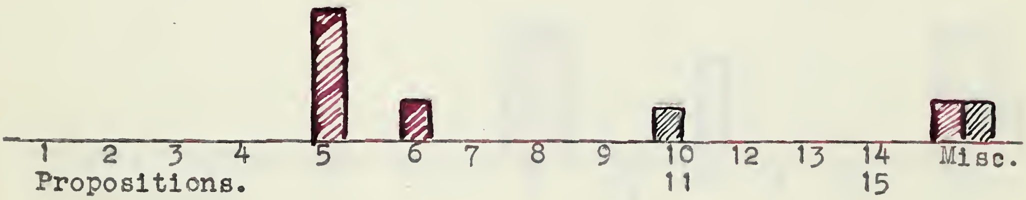


FIGURE VI PROPOSITION VI.

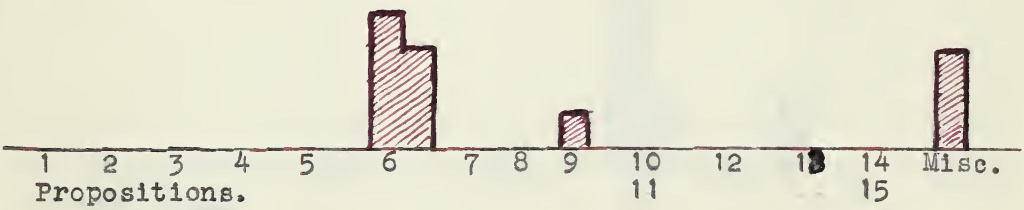
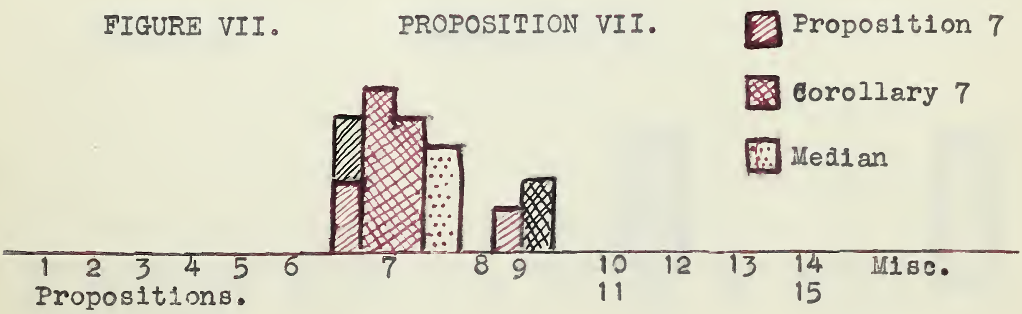


FIGURE VII. PROPOSITION VII.



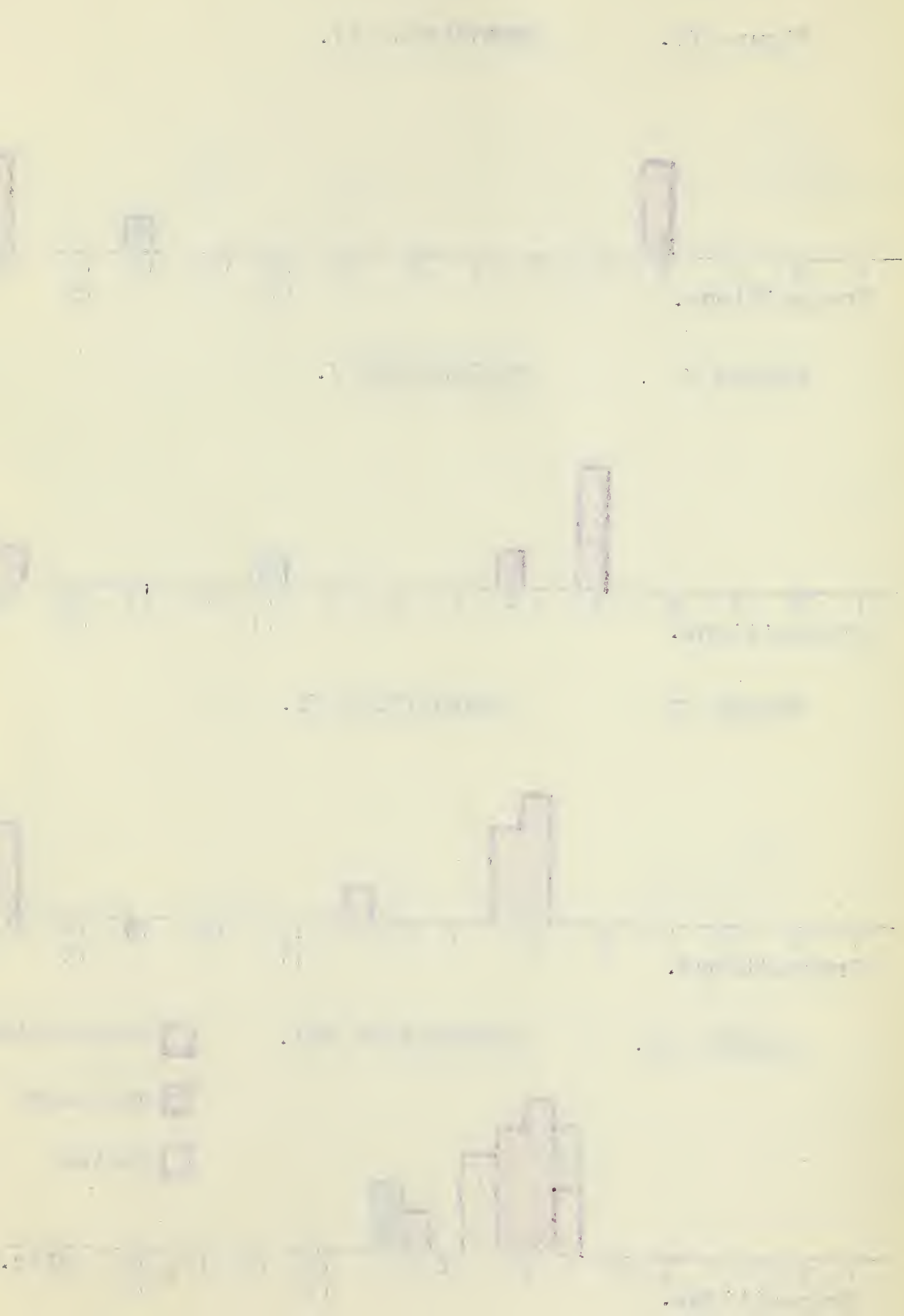


FIGURE VIII PROPOSITION VIII.

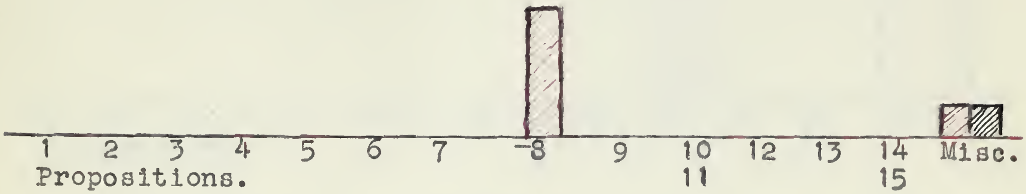


FIGURE IX PROPOSITION IX.

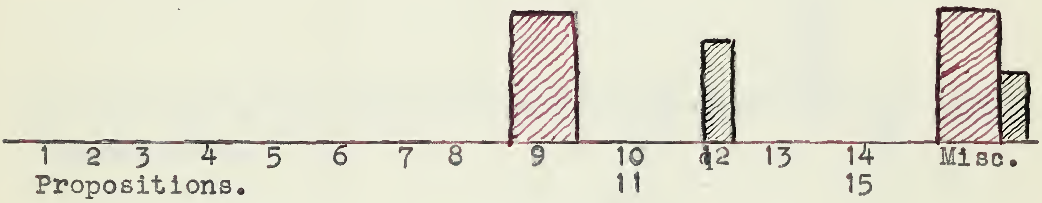


FIGURE X PROPOSITION X

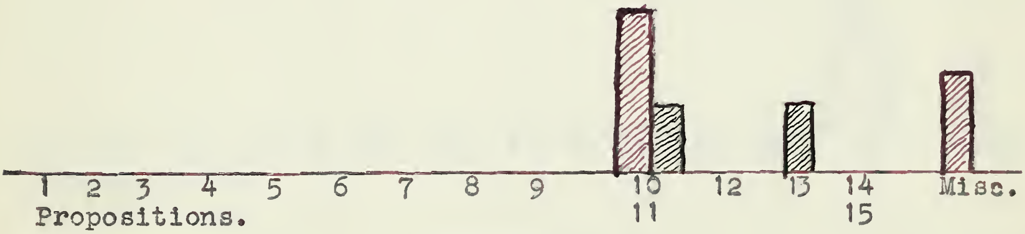


FIGURE XI. PROPOSITION XI.

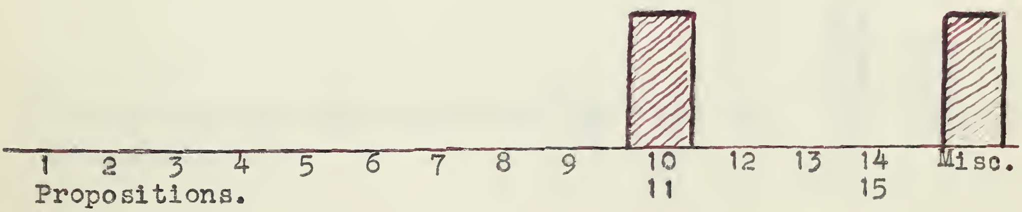


FIGURE XII PROPOSITION XII

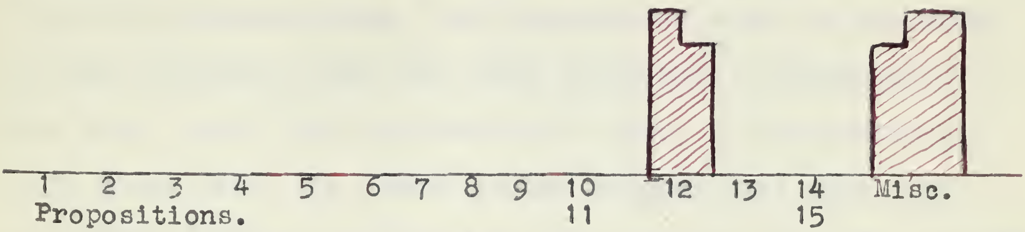


FIGURE XIII. PROPOSITION XIII

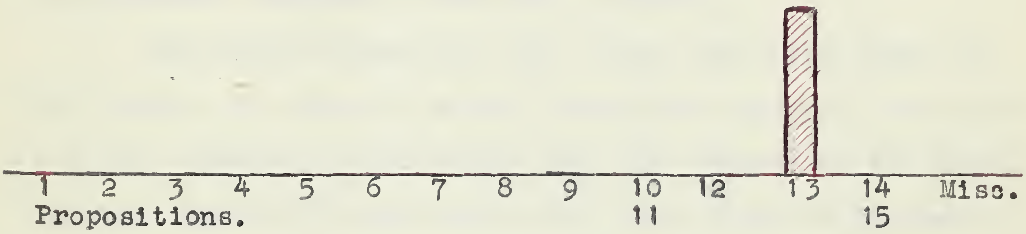


FIGURE XIV. PROPOSITION XIV.

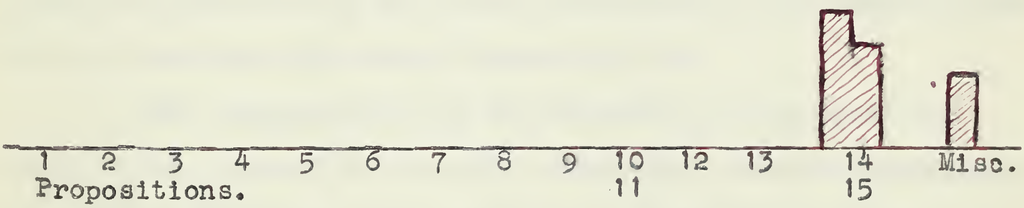
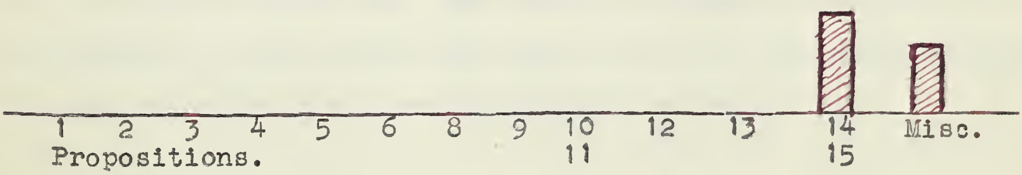


FIGURE XV. PROPOSITION XV.



The last twelve figures, similar to the first three, show by the histograms the amount of drill given to the respective Propositions. The frequency is shown by the area of the histogram. Each unit area represents a frequency of one drill unit. The distribution is shown by the horizontal axis which gives the number of the Propositions which the exercises follow, and listed at the right at the right of each axis, the amount of drill given to each Proposition in the miscellaneous exercise at the end of Book I.

Following Proposition VII, there are three kinds of drill given. One type in on the Proposition proper. The second is on the corollary which deals with the concept of the Right Bisector. The third type deals with the concept of the Median which is presented at the end of Proposition VII. These types are distinguished by the legend of the histograms. The same legend applies to the use of the corollary in the constructions in the exercises following Proposition XI.

The figures show the distribution of the drill and show to some extent the relative frequency for each Proposition. To further facilitate the making of a comparison of the relative frequencies, Table VII sums up the material presented in the figures. The Table is cpmpiled from the data obtained by working the exercises, and shows the amount of major drill, the amount of minor drill and the use made of the constructions in each exercise, for each Proposition in Book I

TABLE VII

A SUMMARY OF THE AMOUNT OF DRILL GIVEN EACH PROPOSITION.

Proposition Book I	Number of Units of Drill Major	Minor	Construction
I	30	15	2
II	11	3	0
III	14	3	2
IV	6	1	0
V	6	0	2
VI	11	0	0
VII	15	4	0
VIII	5	0	1
IX	16	5	0
X	8	4	0
XI	16	0	0
XII	23	0	0
XIII	5	0	0
XIV	9	0	0
XV	5	0	0
Totals	180	35	7

It should be noted in passing that the summary shows only the major use made of the Propositions in Book I. In many cases Propositions are used tacitly in the exercises, without reference being made to them in the work, as in many cases only part of the concept presented by them is used. Again in making constructions, use is made of the Problems, without a note being made to that effect. Also in the above table the three types of drill which follow Proposition VII have been grouped.

In spite of these apparent deficiencies in Table VII, Table VIII lists the Propositions according to the amount of major drill given each, ranking them on this basis.

TABLE VIII.

A RANK LIST OF THE PROPOSITIONS BASED ON THE AMOUNT OF MAJOR DRILL GIVEN EACH IN THE EXERCISES IN BOOK I.

Rank	Proposition	Number of Units of Major Drill
1	I	30
2	XII	23
3	XI	16
4	IX	16
5	VII	15
6	III	14
7	II	11
8	VI	11
9	XIV	9
10	X	8
11	IV	6
12	V	6
13	VIII	5
14	XIII	5
15	XV	5

Table VIII is a summary of the major drill units as shown in the figures, for each Proposition. The Propositions are ranked on the basis of this summary, in that, the Proposition having the greatest number of major drill units is listed first, and the other Propositions follow according to the number of major drill units alloted to each.

In the first part of Chapter III of this thesis the Propositions were classified according to the concept which each general group presented. Table IX shows a summing up of the number of major drill units given to each group, and shows an arrangement, i.e. a rank list according to the number of units given to each. This data is compiled from the figures and Table VIII.

TABLE IX

A RANK LIST OF THE GROUPS OF PROPOSITIONS OF BOOK I BASED ON THE AMOUNT OF MAJOR DRILL GIVEN EACH IN THE EXERCISES.

Groups of Propositions	Major Drill Units
1. Congruence of Triangles	55
2. The Isosceles Triangle	34
3. Inequalities in one Triangle	24
4. Bisectors	21
5. Perpendiculars	16
6. Construction of an angle	16
7. Inequalities in two Triangles	14

Comments on the Exercises.

Before summarizing the data presented by the Tables and Figures of this Chapter, certain comments should be made on the Exercises of Book I.

The classification presented does not show the number of types of exercises which the pupil encounters in this Book. It was noted in working the exercises that a certain number of types are presented. As it seems needless to make an arbitrary list of these here, examples of the recurrence of types of exercises will be given.

As will be noted there is little repetition of the drill given previous Propositions in the drill which is given on each Proposition, particularly in the case of the later Propositions. The reader will note this from the figures which show the distribution of the drill for each Proposition. The repetition of drill and the recurrence of types appears chiefly in the miscellaneous exercise at the end of the Book.

In all, fourteen repetitions are noted in the miscellaneous exercise. These are not all the repetitions in the Book, but are given here as evidence of the fact that there are recurrences of exercises and types of exercises, for drill purposes. Table X presents these recurrences in tabulated form.

TABLE X

RECURRENCES OF EXERCISES AND TYPES OF EXERCISES IN THE MISCELLANEOUS EXERCISE FOLLOWING BOOK I, SHOWING THE NUMBER OF THE EXERCISE IN THE MISCELLANEOUS EXERCISE, AND THE REFERENCE TO THE EXERCISE WHICH IS SIMILAR.

Miscellaneous Exercise Number	Reference to Similar Exercise		
	Page	Number	Proposition of Major Drill Units
24	52	8	I
28	45	2	vertically opposite angles
30	58	2	XII, III
39	58	5	XII
40	58	6	XII
42	47	4	VI, V
47	56	9	XI
58	56	2	X
59	52	10	IX
60	52	10	IX
61	58	7	XII
63	85	8	XII
64	85	8	XII
65	85	8	II

It is difficult to list or display the relative difficulty of the exercises, so this thesis will merely mention the fact that generally speaking the exercises toward the end of each set and toward the end of the Book,

particularly in the miscellaneous exercise, are more difficult than the preceding ones. It is regretted that this statement is not corroborated here.

Analysis of A Problem.

A section appears on pages 66 and 67 of the text giving the pupil instruction in the analysis of a problem. This section is given just before the miscellaneous exercise and no reference is made to it in other parts of the Introduction and Book I.

In this section a suggestion is given as to the manner in which the pupil should proceed in attempting to solve a problem. Although the steps are not enumerated in the text, they are briefly as follows :

1. Begin with the drawing of the given figure or figures.
2. Sketch in the required parts.
3. Make a careful examination to determine the connection between the given parts and the required results.
4. Note the properties of the figure or figures.
5. Draw in lines that may help in finding the solution.

Five examples are given in this section of the method of analysing a problem. The first is carefully set out step by step with each step explained in terms of the accompanying figure. The second and third examples are suggestions of problems similar to the first. The fourth and fifth are more of a suggestive nature and are accompanied by figures. The examples given are types of problems which the pupil encounters

in the miscellaneous exercise. Example 4 is the same as number 41, page 71 of the miscellaneous exercise.

Throughout the exercises a few suggestions are given from time to time to aid the pupil. An example of this is exercise 6, page 49. Also answers are given for the practical problems in many cases, as in number 1,2, page 50.

Some Conclusions.

In the presentation of the data concerning the exercises of Book I, no conclusions have been drawn regarding the relative importance of the Propositions. Table VIII has given a rank list of the Propositions according to the number of major drill units alloted to each. Table IX summed the same data according to groups of Propositions. Before any inferences may be drawn, it seems logical that the same data should be presented according to the constitutent parts of each group, showing the number of major drill units immediately following each Proposition, the number following other Propositions and the number in the Miscellaneous Exercise.

Table XI presents this summary. The data is a summing of the data presented in the figures at the first of this chapter, and is a combination of the Tables which follow these figures. The Table shows the groups of the Propositions ranked as they were in Table IX. Each group is broken up into its parts, showing the number of major drill units for each part, according to the place of these drill units in the Book I. Partially totals are shown in the Table to aid in drawing comparisons.

TABLE XI

DISTRIBUTION OF THE MAJOR DRILL UNITS FOR EACH PROPOSITION FOR GROUPS OF PROPOSITIONS ACCORDING TO OCCURRENCE IN BOOK I.

Group	Proposition	Major Drill Units		
		Immediately Following	Following others.	Miscellane Exercise
1	I	10	7	13
	III	5	2	7
	IV	3	0	3
	XIII	5 (23)	0 (9)	0 (23)
2.	XII	9	0	14
	II	5 (14)	2 (2)	4 (18)
3.	XI	8	0	8
	X	5 (13)	0 (0)	3 (11)
4.	VII	14	1	0
	V	4 (18)	1 (2)	1 (0)
5.	VI	7	1	3
	VIII	4 (11)	0 (1)	1 (4)
6.	IX	8 (8)	0 (0)	8 (8)
7.	XIV	7	0	2
	XV	3 (10)	0 (0)	2 (4)

Based on Tables VIII and IX and the summary of these Table XI, the following inferences are made :

1. The most important group of Propositions in Book I is that dealing with the Congruence of Triangles, which includes Propositions I, III, IV, XIII. Of these Proposition I is the most important.

2. The next important group is that dealing with the Isosceles Triangle, which includes Propositions XII and II. Of these Proposition XII is the more important.

3. The next important group is that dealing with the Inequalities in one Triangle, which includes Propositions X and XI. Of these the more important is Proposition XI.

4. The fourth group in this rank list is that dealing with Bisectors, which includes Propositions V and VII. Of these Proposition VII is more important.

5. The group dealing with Perpendiculars and the group dealing with the Construction of an angle rank next. The group dealing with Perpendiculars includes Propositions VI and VIII. The second includes Proposition IX. Of these apparently Proposition IX is the most important, comparing it with each of the other two Propositions.

6. The group dealing with Inequalities in two Triangles is the least important. This group includes Propositions XIV and XV. Of these Proposition XIV is more important.

7. Although the group dealing with the Congruence of Triangles is the most important, it contains the least important Proposition, that is, Proposition XIII. This Proposition, although relatively difficult is only alloted five major drill units in this classification. These drill exercises appear immediately after the Proposition. No drill units are alloted to it in the miscellaneous exercise. This Proposition was omitted from the Geometry I course in the Province of Alberta during the school year 1929 - 30.

CHAPTER V

SUMMARY AND CONCLUSIONS.

This chapter presents as a summary the conclusions which have been made throughout the thesis concerning the subject matter of the Introduction and Book I of this text. Only the relatively important conclusions are listed here.

1. In comparing the Introduction of this text with the Introduction of Baker's text, it is found that the general concepts presented are the same. The nature of the presentation of the concepts, and the organization or sequence is a little different. For example, this text lists its postulates in Book I, while Baker lists them in his Introduction.

2. This text presents in the Introduction the same general concepts that are presented in the Introduction to Todhunter's Euclid. The presentation is fuller in this text, as it is written for high school pupils.

3. The Introduction is written in accordance with the current scientific and pedagogical thought that a course in Theoretical Geometry should be preceded by a course in Practical Geometry. It has been pointed that the pupil receives many of the concepts in the Introduction by experimentation before the concepts are formally presented to him.

4. The exercises show that the most important concept presented in the Introduction is that dealing with Triangles, with that of Angles ranking second.

5. The second chapter shows that this text includes in the Introduction and in Book I most of the Propositions given by Todhunter Euclid Book I and by Baker in the Introduction

and Book I of his text. Only two of Euclid's Propositions are omitted and only one of Baker's is omitted.

6. The method of presentation of the proofs given here is more like that used by Euclid than that used by Baker. There is much similarity between the proofs of Euclid and the proofs given by this text.

7. The sequence of the Propositions in this text is different from the sequence of the Propositions of Euclid's Book I and Baker's Book I.

8. Chapter IV shows the distribution of the drill given by the exercises of Book I and summarizes the distribution of the major drill units for purposes of comparison.

9. Inferences are made from the data presented that the Congruence of Triangles, given by the four Propositions, is the most important part of Book I. The Isosceles Triangle group ranks next, with the others ranking as, Inequalities in one Triangle, third; Bisectors, fourth; Perpendiculars and Construction of an Angle, fifth; and Inequalities in two Triangles, sixth; respectively.

10. There is a recurrence of exercises and types of exercises in Book I, as shown by the analysis of the exercises in the miscellaneous exercise.

11. A section is included in Book I, prior to the miscellaneous exercise in Book I, which presents to the pupil methods of analysis of Problems. Neither Baker's text, nor Todhunter's Euclid contains such a section. No reference

was found in any other part of this text to the section on Problem Analysis.

12. Certain exercises in the Introduction anticipate by experimentation the Propositions presented in Book I dealing with the Congruence of Triangles.

13. A summary of the cases of Congruence in Triangles appears in Book I, to aid in the consolidation of the material presented.

CHAPTER VI

SOME OPINIONS.

Throughout this thesis, an attempt has been made to give evidence for each conclusion which has been made. In this Chapter, certain opinions are listed, without evidence. These opinions have been formed from a comparison of this text with the other texts, Todhunter's Euclid and Baker; from an analysis of this text; and from experience gained in teaching the Introduction and Book I to a Grade IX class, during the first year in which the text was used in the high school of this province. Briefly the most important of these opinions are as follows :

1. The Introduction of this text is well suited for an experimental presentation of the elementary concepts of a first course in Geometry.

2. The Introduction may be covered satisfactorily in the time alloted to it, in the present Geometry I course. This opinion is based on the exercises of the Introduction, which are about the right number for even the brightest pupils, and which are well chosen with respect to difficulty for the average class.

3. The relative difficulty of the exercises in both Book I and in the Introduction meets the demands of the average class as even the brightest pupils are employed sufficiently during the time alloted to the Introduction and Book I, if they work all the exercises.

4. The more difficult exercises appear toward the end of each set and the difficulty of the exercises increases as

the pupil works toward the end of Book I. Probably the most difficult exercises in the Book appear at the end of the miscellaneous exercise.

5. This text has a better organization from a pedagogical point of view than either Euclid's text or Baker's text. Euclid's text is hardly suited to meet the demands of a Geometry I class, and Baker's text is not as clear or concise as this text. The proofs of this text are more logical and more Euclidean in style than those of Baker's text.

6. The sequence used in this text in both the Introduction and Book I is better than that of either Euclid or Baker from a pedagogical point of view.

7. Any changes in the nature of the proofs have been for the better, in that they make the proofs simpler for the pupil.

8. The section on problem analysis is helpful to the pupil, in aiding him to meet his greatest difficulty in a first course in Geometry. It is regretted that this text does not make greater use of this section, by making references to it.

9. The concepts are presented as needed in Book I. This will be noted by the summary presented in Chapter III. The postulates are given before the Problems in which they are used. It is regretted that this text does not call them postulates.

10. This text has been written primarily for the pupil

in a first course in Geometry. Evidence of this is found in the summary which has been given of the five cases of the Congruence of Triangles. Reference was made to this on page 25 of this thesis. Such a summary aids the pupil in obtaining a wider view of the general nature of the course.